MOTION

Q. What do you Mean by Stream line flow, Laminar flow; Turbulant flow and critical velocity.

Write the Difference b/w Stream Line flow and Turbulant flow.

⇒ Stre

When a liquid of gas flow Such that each particle of Liquid or gas passing to a point follow a path which has been Taken by its previous particle The flow of Liquid or gas is called Streamline flow.

Laminar flow :-

A flow in which the liquid Moves in ~~tains~~ layers is Called a Laminar flow.

Turbulant flow:-

When the Motion of Liquid particle is Random and zig-zag, The flow of Liquid is called the Turbulant flow.

Critical velocity :-

The Max. velocity of a flow of Liquid till it flow Remain

Streamline, it is called critical velocity.

Differnce b/w Streamline & Turbulant flow

Streamline	Turbulant
The velocity of each particle passing a point remains the Same both in magnitude & Direction.	1) The velocity of each particle passing through a point May Variable.
The velocity of flow of Liquid is less than Critical velocity.	2) The velo. of Flow of Liquid is greater than Critical velocity
Path of particles may be either a Straight line or Curved Line.	The path of particle is irregular and zig-zag.
At each point the pressure and Density is constant	At each point the pres. and Density of Liquid do not Remain Constant.

What do you Mean by viscosity and viscous force. Write the effect of Temp and pressure on viscosity

⇒ The property of liquid by virtue of which the tendecy to oppose the Relative motion b/w its Different Layers is called viscosity.

Viscus Force :- The force of friction opposing the Relative Motion b/w the two Successive Layers of Liquid is called viscus force.

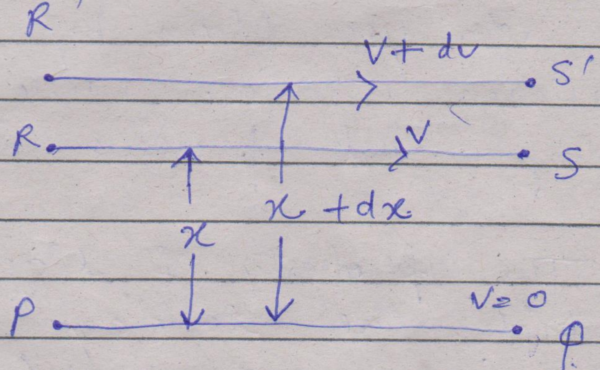
Effect of Temperature on

Viscosity ~~of~~ of a liquid Decreases with the increase in Temp. because the K.E of Liquid Molecule Increase with the Increase in Temp. Hence the cohesive force b/w them Decrease. viscosity of gas increases with the increase in Temp. ~~due to~~ momentum transfer due to transfer of gas molecules increase with the increase in Temp.

EFFECT of Pressure on Viscosity :-

⇒ viscosity of a Liquid increases with the increase in pressure. but the viscosity of gas Remain nearly unaffected increase in pressure.

Ques. Write the Newton's Law for the viscous force And Define the cofficient of viscosity with the help of it.



Velocity gradient :-

The change in velocity of 2 layers of Liquid separated by unit Distance is called the velocity gradient.

consider a liquid flowing ~~a~~ in a Stream-line condition on a fixed horizontal Surface PQ. Let x and $x+dx$ respectively be the vertical heights of the Liquid Layer's R, S and R' S' from the fixed Horizontal Surface PQ. The velocity of flow of these Layers be v and $v+dv$ respectively. Then, velocity gradient equals to $\frac{dv}{dx}$

$$\text{velocity gradient} = \frac{dv}{dx}.$$

velocity gradient is a vector Quantity.

Acc. to the Newton the viscous Force "F" b/w the 2 Successive Layers of Liquid Depends upon the following Factors.

1) Surface Area of Layer in Contact :—
The viscous Force F is $\propto$ to the Surface Area (A)

i.e., $F \propto A$

2) Velocity gradient :-
The viscous force F is $\propto$ to the velocity gradient.

$$F \propto \frac{dv}{dx}$$

$$\Rightarrow F \propto A\frac{dv}{dx}$$

$$\text{or} \quad \boxed{F = -\eta A\frac{dv}{dx}}$$

where; η is a constant it is called the cofficient of viscosity. And -ve Sign indicates that the viscous Force is opposing to the Direction of flow of Liquid.

if $A = 1$ $\frac{dv}{dx} = 1$

then ; $F = \eta$ (Numerically)

The cofficent of viscosity of Liquid is Numerically equal to the external force Required to Mantain Unit velocity gradient b/w two Successive Layer of unit Surface area.

$$\eta = \frac{F \cdot dx}{A \cdot dv}$$

$$\eta = \frac{N \cdot \cancel{m}}{m^2 \cdot \cancel{m}/s}$$

$$\eta \Rightarrow N s / m^2$$

$$\eta \rightarrow \frac{kg \; m s^{-2} \cdot s}{m^2}$$

$$\eta \rightarrow kg \; m^{-1} s^{-1}$$

C.G.S. → $\eta \Rightarrow g \; cm^{-1} s^{-1}$ [poise]

Dimensional formula = $[M L^{-1} T^{-1}]$

Ques. What do you Mean by Reynold's No. What are the Factor affecting the Critical velocity. Derive the expression for the Critical velocity?

⇒ Reynold's No. is a pure No. which Determines the Nature of flow of Liquid through a pipe.

⇒ Significance of Reynold's No :-

1.) If the value of Reynolds No. lies b/w 0 and 2000, the flow of viscous Liquid in tube is Streamline.

2.) If the value of Reynolds No. is in b/w 2000 and 3000, the flow of Liquid in tube is Temperary. And it can Change from the Stream Line flow to the Turbulant flow.

3.) If the value of Reynolds No. is more than 3000 the flow of liquid in the tube is Turbulant.

⇒ Factors Affecting critical velocity :-

A/c to Reynold's for a liquid flowing in horizontal capillary the critical velocity depends on the following factors.

1.) Dependence on density of Liquid :-

The critical velocity V_c is inversly proportional to the density ρ of the liquid

i.e., $V_c \propto \frac{1}{\rho}$

2. Dependence on Radius of capillary.

The critical velocity V_c is inversly to the radius "r" of the capillary.

i.e., $V_c \propto \frac{1}{r}$

on viscosity of liquid.

⇒ The critical velocity v_c is directly proportional to the cofficient of viscosity η of the liquid.

$$v_c \propto \eta$$

$$v_c \propto \frac{\eta}{\rho r}$$

$$\boxed{v_c = \frac{N_R\,\eta}{\rho r}}$$

hence N_R is a constant which is called Reynolds Number.

* Derivation by Dimensional Method :-

Since the critical velocity v_c depends on density ρ of the liquid Radius "r" of the capillary and cofficient of viscosity η of the liquid,

$$v_c \propto \eta^a \rho^b r^c$$

$$v_c = K \eta^a \rho^b r^c \quad \text{—(1)}$$

$$[M^0 L T^{-1}] = [M L^{-1} T^{-1}]^a [M L^{-3}]^b [L]^c$$

$$[M^0 L T^{-1}] = [M^{a+b} L^{-a-3b+c} T^{-a}]$$

$-a = -1$

$a = 1$

$\Rightarrow a + b = 0$

$1 + b = 0$

$b = -1$

$\Rightarrow -a - 3b + c = 1$

$-1 - 3(-1) + c = 1$

$-1 + 3 + c = 1$

$2 + c = 1 \Rightarrow c = -1$

from ①

$$v_c = k\eta\, \rho^{-1} r^{-1}$$

$$v_c = \frac{k\eta}{\rho r}$$

$$\boxed{v_c = \frac{N_R\, \eta}{\rho r}}$$

Que. Write the Stoke's Law and Derive it by Dimensional Method.

A/c to Stoke's Law if a small spherical body fall down with a constant velocity in a perfectly homogeneous viscous medium the viscous force act in the opposite direction whose magnitude depends ~~upon~~ on the radius of body its speed and viscosity of medium.

Let a spherical body of Radius r be fall down with a constant velocity v in a perfectly homogeneous viscous medium of cofficient of viscosity η the magnitude of Force F.

proportional to radius r of the body i.e.,

$F \propto r$

2.) Directly proportional to cofficient of viscosity η of the medium.

$F \propto \eta$

3.) Directly proportional to velocity v of the body.

$F \propto v$

$F \propto r\eta v$

$$\boxed{F = 6\pi r\eta v}$$

<u>Derivation by Dimensional Method</u> :-

$$F \propto \eta^a r^b V^c$$

$$F = K\eta^a r^b V^c \quad (1)$$

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^a [L^b] [LT^{-1}]^c$$

$$[MLT^{-2}] = [M^a L^{-a+b+c} T^{-a-c}]$$

$a = 1$

$\Rightarrow -a - c = -2$

$a + c = 2$

$c = 2 - 1$

$c = 1$

$\Rightarrow -a + b + c = 1$

$\Rightarrow -1 + b + 1 = 1$

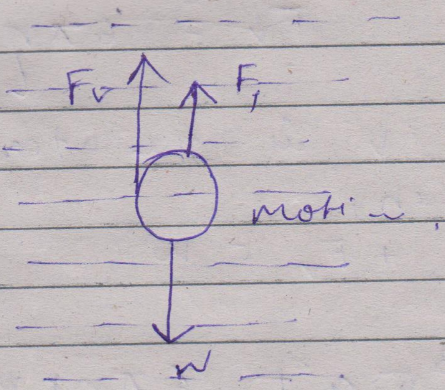
$b = 1$

$$F = K\eta^{1} r^{1} v^{1}$$

$$\boxed{F = K\eta r v}$$

$$\boxed{F = 6\pi\eta r v}$$

Que. What do you mean by Terminal velocity find the expression for it.



It is the maximum constant velocity acquired by the body while falling freely in viscus medium.

Let a small spherical body be falling freely in a viscus Liquid under gravity. Let cofficient of viscosity of liquid be η, density of liquid be σ. Radius of body be r and density of material of the body be ρ.

The following three forces act on the body.

i) weight of body.

$$W = mg = V\rho g.$$

$$W = \frac{4}{3}\pi r^3 \rho g \downarrow$$

$$W = \frac{4}{3}\pi r^3 \rho g \downarrow$$

(ii) upthrust due to Medium

$$F_T = V\sigma g$$

$$F_T = \frac{4}{3}\pi r^3 \sigma \uparrow$$

iii) viscous force

$$F_v = 6\pi\eta r v \uparrow$$

If v is the terminal velocity then,

$$F_T + F_v = W$$

$$\frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g$$

$$6\pi\eta r v = \frac{4}{3}\pi r^2 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

$$6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

$$v = \frac{\frac{4}{3}\pi r^3 (\rho - \sigma) g}{\frac{6}{3}\pi\eta r}$$

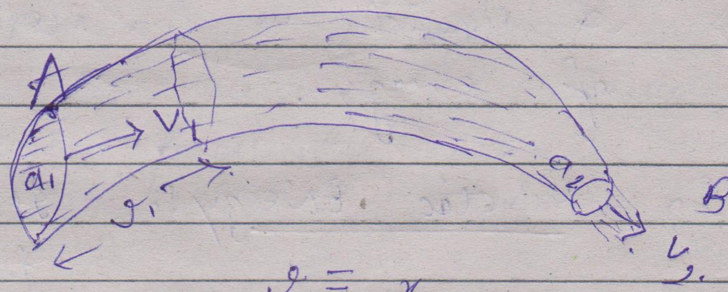
$$\boxed{v = \frac{2r^2(\rho - \sigma) g}{9\eta}}$$

Q. Derive the eq. of continuity.

⇒ "Acc. to the eq. of continuity when an ideal liquid flows in a stream line condition to a tube of non-uniform cross-section. And each sec. of the tube, ~~area~~ product of the area of cross-section and velocity of liquid is constant"

If at any part of the tube, the ar. of cross section is "a" and velo. of flow of liquid is "v" then by principle of continuity.

$$\boxed{av = \text{constant}}$$



$v = \frac{x}{t}$

$t = 1$; $v = x$.

$m_1 = a_1 v_1 \rho$

Consider an ideal liquid is in stream-line flow to a tube of non uniform cross-sectional.

Let $a_1 a_2$ = area of cross section of tube at a and v respectively. ~~$v_1 v_2$ =~~

$v_1 v_2$ = velocity of flow of liquid at a and b respectively.

ρ = Density of liquid.

Vol. of liquid entering / sec at a = $a_1 v_1$

Mass of Liquid entering / sec at A = $m_1 = a_1 v_1 \rho$

Similarly,

mass of liquid emerging out per sec at B = $m_2 = a_2 v_2 \rho$.

Since no liquid collects at any point of the tube therefore

$m_1 = m_2$.

$$a_1 v_1 \rho = a_2 v_2 \rho$$

$$a_1 v_1 = a_2 v_2$$

i.e., $\boxed{av = \text{constant}}$.

Ques.:- Explain the different form of energy of flowing liquid.

⇒ A liquid is in Motion has 3 types of energy.

(i) Kinetic Energy:- It is the energy possessed by a liquid by virtue of its Motion.

Let "v" be the velocity of liquid of mass "m" Density ρ and vol. V. ~~per unit volume~~.

$$K.E = \frac{1}{2} m v^2$$

K.E of liquid per unit volume.

$$1 = \frac{1}{2} \rho v^2$$

⇒ **Potential energy :-**

It is the energy possessed by liquid by virtue of its height or position above the surface of earth.

Let m be the mass of liquid and height "h" above the surface of earth.

V be the volume and ρ be the density

Hence;

P.E of the liquid $= mgh$

P.E / unit vol. of the liquid $= \frac{mgh}{V}$

$= \rho g h.$

⇒ **Pressure Energy :-** It is the energy possessed by a liquid by virtue of its pressure it is equal to the work done in pushing the liquid in the vessel against pressure without imparting any velocity to it.

If pressure P - newton / m^2 acts on an Area "A" - m^2 of a flowing liquid and the liquid displaced by x - meter,

Pressure energy = work done by the liquid in displacement "x".

work done $= F \times x$

$= P \times A \times x$

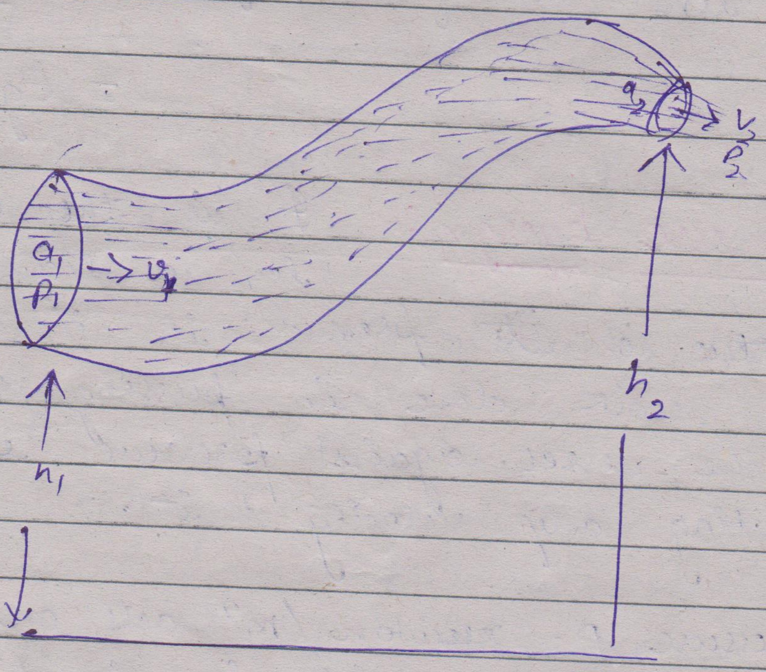
$= P \cdot V$ $[Ax = V = \text{volume}]$

pressure energy / unit vol. of liquid $= \frac{PV}{V} = P$

Que: State And prove the Bernouli's theorem

Acc. to Bernouli's theorem when an ideal liquid flows from one point to the other in a Stream-Line condition without any frictional or Resistive force, the total energy of per unit volume of liquid remains constant at each point of its flow i.e.,

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$



Consider a tube AB of Non uniform cross-Section through which an ideal liquid is in Streamline flow.

Let P_1 = applied pressure on liquid at A

P_2 = pressure at the end B.

$a_1\, a_2$ = Area of cross-section of the tube at A and B respectively.

$v_1\, v_2$ = velocity of flow of liquid at A and B respectively.

$h_1\, h_2$ = height of the Section A and B from the ground respectively.

ρ = Density of the liquid

Force on the liquid at Section A $= P_1 a_1$

work done / sec on the liquid by the pressure energy at Section A $= P_1 a_1 v_1$

Similarly;

work done / sec by the liquid against the pressure energy at Section B $= P_2 a_2 v_2$

Net work done / sec on the liquid by pressure energy in Moving the liquid from Section A to B $= P_1 a_1 v_1 - P_2 a_2 v_2$

$$= P_1 \frac{m}{\rho} - P_2 \frac{m}{\rho}$$

$$= [P_1 - P_2] \frac{m}{\rho} \quad (1)$$

Change in K.E / Increase in K.E / per Second of liquid from A to B =

$$\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \quad (2)$$

Change in P.E / per Second of liquid from A to B $= mgh_2 - mgh_1$ — (3)

Acc. to work energy principle:-

work done / per second on the liquid by the pressure energy = Increase in K.E + Increase in P.E.

$$\Rightarrow (P_1 - P_2) \frac{m}{\rho} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + mgh_2 - mgh_1$$

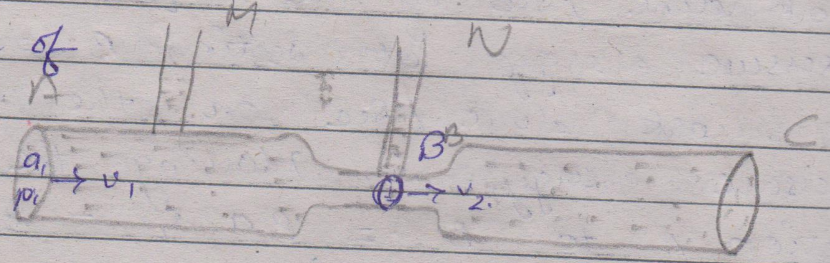
$$\Rightarrow P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 + \rho g h_2 - \rho g h_1$$

$$\Rightarrow P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

i.e., $\boxed{P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}}$!—

Q What do you Mean by Ventrimeter. Derive the expression for rate of flow of liquid.

It is a device based on Bernoulli's theorem which is used to find the rate of flow of liquid in a Tube.



Construction:- It consist of a Tube A, B and C the middle part B of the tube is thin then the part A and C. It is joint with 2 vertical tube and end in Section A and B the tube AC is connected with the Tube in which the rate of flow of liquid is to be determined.

working:- Let the area of cross section of the tube at A and B be a_1 and a_2 respectively $a_1 > a_2$ when liquid lose in the Tube the velocity of flow at B is greater then at C if v_1 and v_2 be the velocity of flow of liquid at A and B respectively. then by equation of continuity $a_1 v_1 = a_2 v_2$ Since $a_2 < a_1$ therefore $v_2 > v_1$ the velocity of flow of liquid at B is greater than that at A therefore by Bernoulli's theorem the pressure of liquid at B is less then at A due to which the height of liquid column in Tube M is greater then the height of liquid column in tube N.

Let h be the difference in height of these column and ρ be the density

$$P_1 - P_2 = h\rho g \quad (1)$$

by bernoulli's theorem

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$h\rho g = \frac{1}{2}\rho v_1^2\left[\frac{v_2^2}{v_1^2} - 1\right]$$

$$2gh = v_1^2\left[\frac{a_1^2}{a_2^2} - 1\right]$$

$$2gh = v_1^2\left[\frac{a_1^2 - a_2^2}{a_2^2}\right]$$

$$v_1^2 = a_1^2 \cdot \frac{2gh}{a_1^2 - a_2^2}$$

$$\boxed{v_1 = a_2 \cdot \sqrt{\frac{2gh}{a_1^2 - a_2^2}}}$$

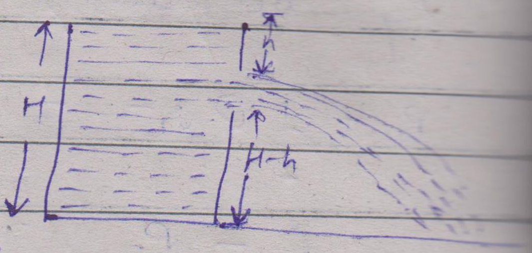
The volume of liquid flowing per second.

$$V = a_1 v_1$$

$$\boxed{V = a_1 a_2 \cdot \sqrt{\frac{2gh}{a_1^2 - a_2^2}}}$$

Ques :- State And prove the Torricelli's theorem.

A/C to the Toricelli's theorem the velocity of ~~off~~ efflux of a liquid through in orifice in a vessel is = velocity free surface of liquid to that orifice



consider a vessel containing a liquid up to a height H. Let there be an orifice S and at depth h below the free surface of liquid at the free surface of liquid at v and if the ρ is the Density of liquid then P.E per unit volume of the liquid is ρgh.

Just outside the orifice the pressure is, the K.E of per unit volume of liquid is $\frac{1}{2}\rho v^2$, where v is the velocity of efflux of liquid and P.E of per unit volume of liquid $= \rho g(H-h)$.

By Bernoulli's theorem total energy of per unit volume of liquid at free surface = total energy of unit of volume of liquid just outside the orifice.

$$P + 0 + \rho gH = P + \frac{1}{2}\rho v^2 + \rho g(H-h)$$

$$\rho gH = \frac{1}{2}\rho v^2 + \rho gH - \rho gh$$

$$0 = \frac{1}{2}\rho v^2 - \rho gh$$

$$\frac{1}{2}\rho v^2 = \rho gh$$

$$v^2 = 2gh$$

$$\boxed{v = \sqrt{2gh}} \quad (1)$$

Suppose "v" be the velocity acquired by liquid in free fall from free surface of liquid to the oriffice then by 3rd equation of Motion:-

$$v^2 = u^2 + 2gh$$

But $u = 0$

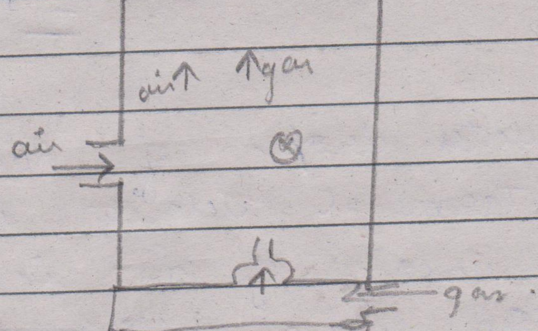
$$v'^2 = 2gh$$

$$\boxed{v' = \sqrt{2gh}} \quad -(2)$$

from eq. (1) and (2)

$$\boxed{v = v'}$$

thus; the velocity of efflux of a liquid through an oriffice is equals to the velocity acquired by it in free fall from the free surface to that oriffice.



The diameter of a capillary is 2mm if Reynolds No. of the tube is 1000 and cofficient of viscosity for the water is 0.01 poise. Calculate the Maximum